

Theoretical Distribution

Distribution of total probability on the basis of a random variable which can be discrete or continuous.

So we can say that Theoretical Distribution is a mathematical function that describes all the possible values of a random variable & their probabilities

If Random variable is discrete then Probability distribution is known as probability mass function

eg

x_i	p_i
1	0.2
2	0.4
3	0.3
4	0.1
	1

X is discrete Random variable
like: No of accidents
, No of goals in a football match
& No of Defective bulbs

In this chapter we will study about two discrete prob. Distribution

1) Binomial Distribution

2) Poisson Distribution

If Random variable is continuous
Then probab. Distribution is
known as probab. Density function
of x_i : Height, Age, wages, salary.

<u>CI</u>	<u>P_i</u>
0-10	0.2
10-20	0.5
20-30	0.1
30-40	0.2
	<u>1</u>

In this chapter
we will study
only one continuous
probab. Distribution
which is

"Normal Distribution"

Binomial Distribution

→ Also known as Bernoulli Trials

→ According To Bernoulli

→ If an experiment is performed ' n ' times

Then $n = \text{No. of Trials}$

→ No of Trials is a finite positive integer

→ outcomes of each trials are categorized as success (p) & failure (q)

→ In each trial prob. of success & failure remain same (Trials are independent)

→ $p + q = 1$ & $q = 1 - p$

→ If x is a random variable then $x = 0, 1, 2, 3, \dots, n$

$$\left[\begin{array}{l} x \sim B(n, p) \\ P(x=r) = {}^n C_r p^r q^{n-r} \\ \text{for } r = 0, 1, 2, 3, \dots, n \end{array} \right.$$

$$P(x=0) + P(x=1) + \dots + P(x=n) = 1$$

→ Binomial Distribution is biparametric, There are two parameters n & p

$$\rightarrow \text{mean}(\mu) = np$$

$$\rightarrow \text{variance}(\sigma^2) = npq$$

$$\rightarrow \text{S.D.}(\sigma) = \sqrt{npq}$$

$$\rightarrow \text{maximum variance} = \frac{n}{4}$$

$$\text{when } p = q = \frac{1}{2}$$

$\rightarrow$ Additive property

$$\text{If } X \sim B(n_1, p)$$

$$Y \sim B(n_2, p)$$

$$\text{Then } X + Y \sim B(n_1 + n_2, p)$$

→ mode of Binomial Distribution
Depends on the value
of $(n+1)p$.

[If $(n+1)p$ is non integer
Then mode = $[(n+1)p]$ greatest
integer

[If $(n+1)p$ is integer
Then there are two modes
First mode = $(n+1)p$
Second mode = $(n+1)p - 1$

→ This used when 'n' is small
& p is not small

Poisson Distribution

for discrete random variable

- used when prob. of success in a small time interval is very small.
- n is big & p is small
- mean = $m = np$
- Poisson Distribution is uniparametric
 m is the only parameter
- for Random variable X
 $X \sim P(m)$

$$P(X=x) = \frac{e^{-m} m^x}{x!}$$

for $x = 0, 1, 2, 3, \dots, \infty$

$$\rightarrow P(X=0) + P(X=1) + P(X=2) + \dots = 1$$

$$\rightarrow e = 2.7183$$

$$\rightarrow e^x = A_L(x \times 0.4343)$$

$$\rightarrow \text{mean} = m = np$$

$$\rightarrow \text{variance} = m = np$$

$$\rightarrow \text{S.D.} = \sqrt{m}$$

$\rightarrow$

$$\text{mode} = \begin{cases} [m] & \text{if } m \text{ is non integer} \\ \text{integral part} \\ m \text{ \& } m-1 & \text{if } m \text{ is integer} \end{cases}$$

$\rightarrow$ Additive property

if $x \sim P(m_1)$ & $y \sim P(m_2)$

Then $x + y \sim P(m_1 + m_2)$

Normal Distribution

Also known as Gaussian Distribution

most important & universally
accepted continuous prob.

Distribution function

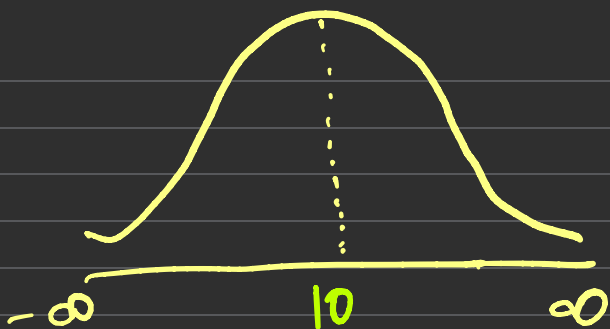
It is a biparametric Distribution

mean (μ) & variance (σ^2)

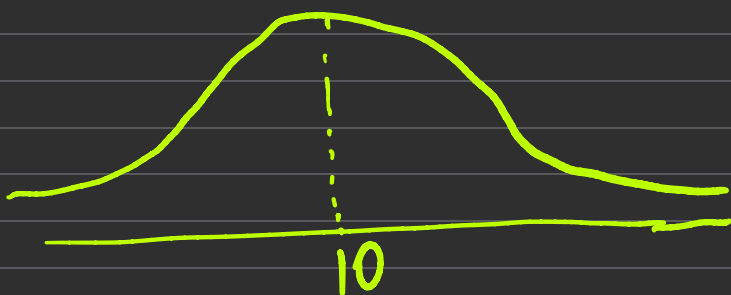
are two parameters
which decides the shape of
normal Distribution curve

$\mu \Rightarrow$ It will tell central value

$\sigma^2 \Rightarrow$ It decides the spread



$$\mu = 10$$
$$\sigma^2 = 2$$

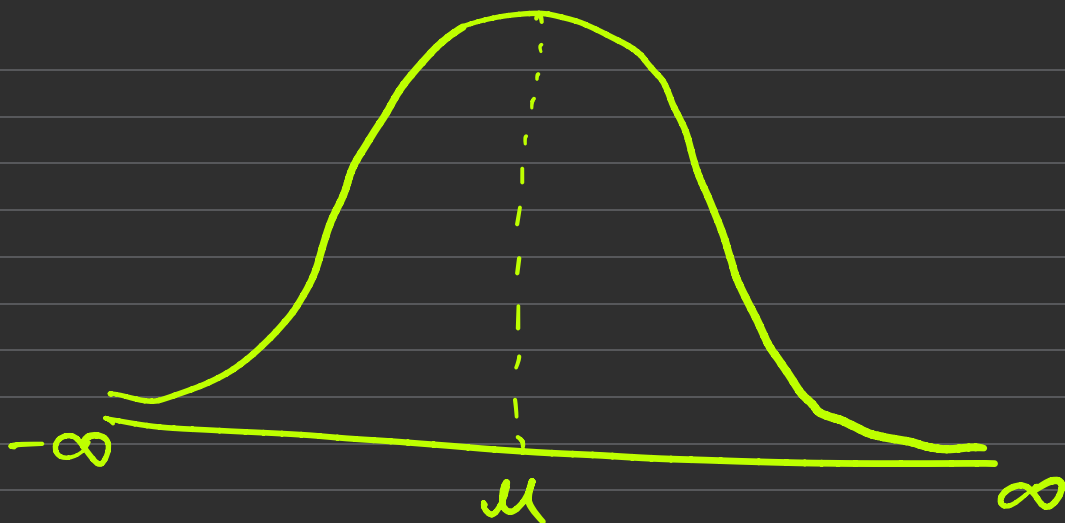


$$\mu = 10$$
$$\sigma^2 = 5$$

$$X \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2}$$

for all $-\infty < x < \infty$



→ Normal curve is a symmetrical curve

→ Skewness = zero

→ Area under this curve is taken 1

$$\int_{-\infty}^{\infty} f(x) = 1$$

$$\int_{-\infty}^{\mu} f(x) = 0.5$$

$$\int_{\mu}^{\infty} f(x) = 0.5$$

→ Both tails of the curve never touches horizontal axis (X-AXIS)

→ In normal Distribution
 $\text{mean} = \text{median} = \text{mode}$

→ mean Deviation = 0.80

→ Quartile Deviation (QD) = 0.6750

$$Q_1 = \mu - 0.6750$$

$$Q_3 = \mu + 0.6750$$

$$\begin{array}{l} \text{median} \\ (\text{mean}) \\ (\text{mode}) \end{array} = \frac{Q_3 + Q_1}{2}$$

→ Point of inflexion
 $\mu - \sigma$ & $\mu + \sigma$

→ Additive Property

$$\text{If } x \sim N(\mu_1, \sigma_1^2)$$

$$\& y \sim N(\mu_2, \sigma_2^2)$$

$$\text{Then } x + y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$$

→ Standard normal variate

$$\text{If } \mu = 0 \& \sigma = 1$$

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2}$$

This is known as standard normal variate

$$z = \frac{x - \mu}{\sigma}$$

for standard normal variate

$$\mu = 0 \text{ \& } \sigma = 1$$

$$\rightarrow \text{mean} = \text{median} = \text{mode} = 0$$

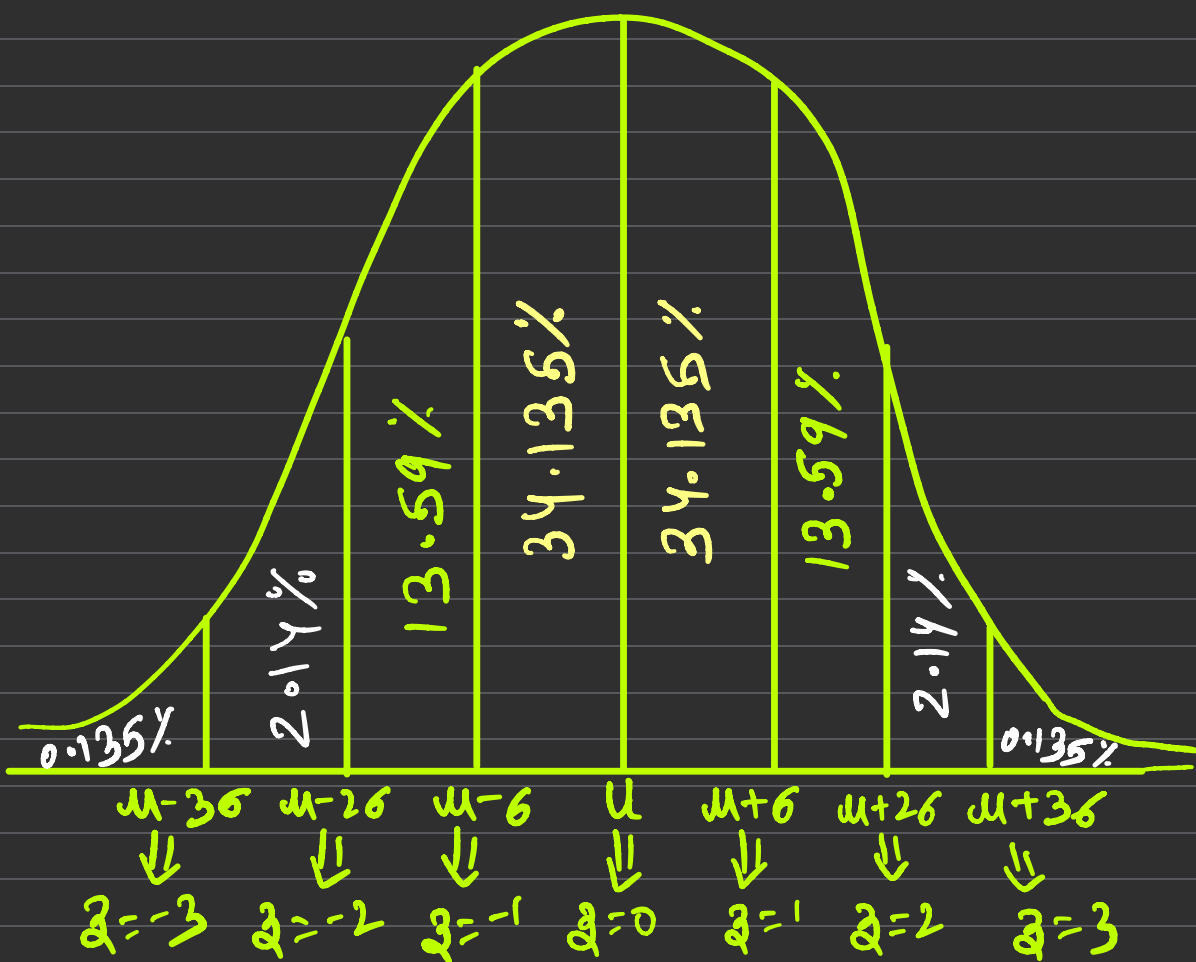
$$\rightarrow \text{MD} = 0.8\sigma = 0.8$$

$$\rightarrow \text{Q.D} = 0.675\sigma = 0.675$$

$$Q_1 = -0.675$$

$$Q_3 = 0.675$$

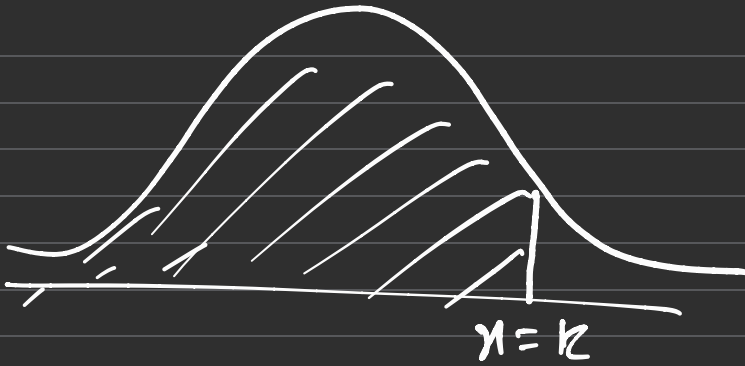
$$\rightarrow \text{Point of inflexion} = -1 \text{ \& } 1$$



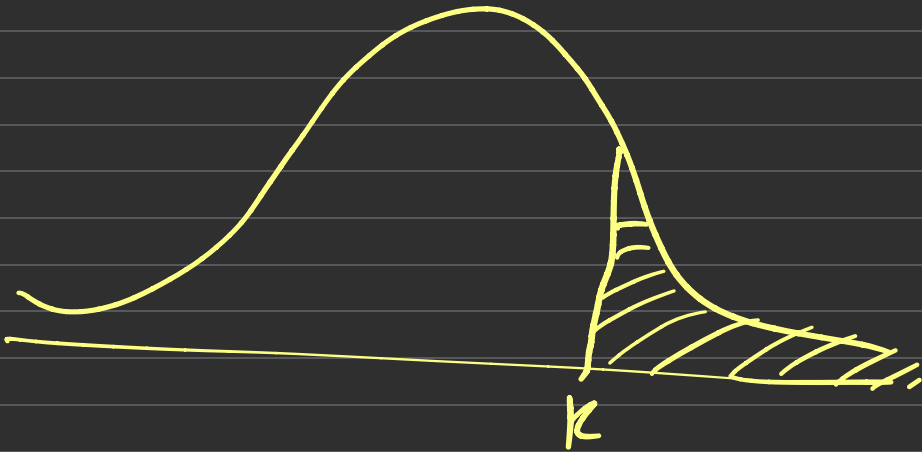
$$P(0 < z < 1) = \frac{34.135}{100} = 0.3413$$

$$P(-1 < z < 1) = \frac{68.27}{100} = 0.6827$$

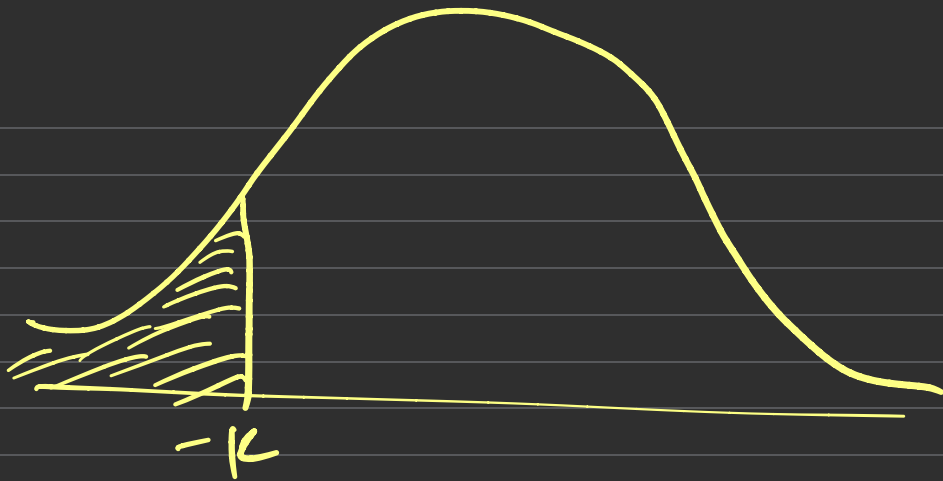
$$P(\mu < z < \infty) = \frac{50}{100} = 0.50$$



$$P(X \leq k) = \Phi(k)$$



$$\begin{aligned} P(X > k) &= 1 - P(X \leq k) \\ &= 1 - \Phi(k) \end{aligned}$$



$$P(x < -k) = 1 - P(x < k)$$

$$\phi(-k) = 1 - \phi(k)$$